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关于Mycielski图补图的一些指标的结果

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Mycielski Schultz Schultz Mycielski Lanzhou

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$$S^*(G) = \sum_{\{v_i,v_j\} \quad V(G)} d_G(v_i,v_j)\left(d_G(v_i)\,d_G(v_j)\right)$$

Vukičevič G Lanzhou [10]

$$Lz(G) = \sum_{u \quad V(G)} \bar{d}_u d^2{}_u$$

d_u $\bar{d}_u$ G u G $\bar{G}$ u Lanzhou Lz G

$(n - 1)M_1(G) - F(G)$, n G M_1 G G Zagreb F G G Forgotten

[11-13]

$$F(G) = \sum_{v_i \quad V(G)} d_G^3(v_i) = \sum_{v_i v_j \quad E(G)} d_G^2(v_i) + d_G^2(v_j)$$

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Mycielski Lanzhou

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$G = (V(G), E(\quad))$

$$(i)\, d_{\mu(G)}(u,v)=\begin{cases}2, & u=x_i, v=x_j\\ d_G(v_i,v_j), & u=v_i, v=v_j, d_G(v_i,v_j)\le 3\\ 4, & u=v_i, v=v_j, d_G(v_i,v_j)\ge 4\end{cases}$$

$$(ii)\, d_{\mu(G)}(u,v)=\begin{cases}2, & u=v_i, v=x_j, i=j\\ d_G(v_i,v_j), & u=v_i, v=x_j, i\ne j, d_G(v_i,v_j)\le 2\\ 3, & u=v_i, v=x_j, i\ne j, d_G(v_i,v_j)\ge 3\end{cases}$$

$$(iii)\, d_{\mu(G)}(u,v)=\begin{cases}1, & u=x_i, v=z\\ 2, & u=v_i, v=z\end{cases}$$

$$(iv)\, d_{\bar{\mu}(G)}(u,v)=\begin{cases}1, & u=v_i, v=v_j, d_G(v_i,v_j)>1, v_iv_j\quad E(G)\\ 2, & u=v_i, v=v_j, d_G(v_i,v_j)=1, v_iv_j\quad E(G)\\ 1, & u=x_i, v=x_j\end{cases}$$

$$(v)\, d_{\bar{\mu}(G)}(u,v)=\begin{cases}1, & u=v_i, v=x_j, i=j\\ 1, & u=v_i, v=x_j, i\ne j, d_G(v_i,v_j)>1\\ 2, & u=v_i, v=x_j, i\ne j, d_G(v_i,v_j)\ge 1\end{cases}$$

$$(vi)\, d_{\bar{\mu}(G)}(u,v)=\begin{cases}1, & u=v_i, v=z\\ 2, & u=x_i, v=z\end{cases}$$

$\mu(G)$ u v 4 $\bar{\mu}(G)$ u v 2.

2.1 Mycielski Schultz Schultz

1 G n m Mycielski $\bar{\mu}(G)$ Schultz

$S(\bar{\mu}(G)) = 8n^3 + 3n^2 - n - 4m - 5M_1(G)$.

$\mu(G)$ u v 1 $\{v_i, v_j\}$ G 2 $\{x_i, x_j\}$ G 3 $\{v_i, x_j\}$ G G 4 $\{v_i, z\}$ G 5 $\{x_i, z\}$ G

1 u v 1 Schultz

$$\begin{aligned}
&\sum_{\{v_i,v_j\}\quad V(G)} d_{\bar{\mu}(G)}(v_i,v_j)\left(d_{\bar{\mu}(G)}(v_i)+d_{\bar{\mu}(G)}(v_j)\right)\\
&=\sum_{\substack{d_G(v_i,v_j)=1\\ v_iv_j\quad E(G)}} d_{\bar{\mu}(G)}(v_i,v_j)\left(d_{\bar{\mu}(G)}(v_i)+d_{\bar{\mu}(G)}(v_j)\right)+\sum_{\substack{d_G(v_i,v_j)>1\\ v_iv_j\quad E(G)}} d_{\bar{\mu}(G)}(v_i,v_j)\left(d_{\bar{\mu}(G)}(v_i)+d_{\bar{\mu}(G)}(v_j)\right)\\
&=\sum_{\substack{d_G(v_i,v_j)=1\\ v_iv_j\quad E(G)}} 2\left[2n-2d_G(v_i)+2n-2d_G(v_j)\right]+\sum_{\substack{d_G(v_i,v_j)>1\\ v_iv_j\quad E(G)}}\left[2n-2d_G(v_i)+2n-2d_G(v_j)\right]\\
&=\sum_{v_iv_j\quad E(G)} 2\left(4n-2\left(d_G(v_i)+d_G(v_j)\right)\right)+\sum_{v_iv_j\quad E(G)}\left(4n-2\left(d_G(v_i)+d_G(v_j)\right)\right)\\
&=2n^3-2n^2+4m-2M_1(G)
\end{aligned}$$

2 u v 2 Schultz

$$\begin{aligned}
&\sum_{\{x_i,x_j\}\quad v(X)} d_{\bar{\mu}(G)}(x_i,x_j)\left(d_{\bar{\mu}(G)}(x_i)+d_{\bar{\mu}(G)}(x_j)\right)\\
&=\sum_{\{x_i,x_j\}\quad v(X)}\left[2n-1-d_G(v_i)\right]+\left[2n-1-d_G(v_j)\right]
\end{aligned}$$

$$= \sum_{\{x_i, x_j\} \quad v(X)} 2(2n - 1) - \left(d_G(v_i) + d_G(v_j)\right)$$

$$= 2n^3 - 3n^2 + (1 - 2m)n + 2m$$

3 u v 3 Schultz

$i = j$

$$\sum_{v_i \quad v(G), x_j \quad v(x)} d_{\mu(G)}(v_i, x_j)\left(d_{\mu(G)}(v_i) + d_{\mu(G)}(x_j)\right) = \sum_{v_i \quad v(G)} 1\left(2n - 2d_G(v_i) + 2n - 1 - d_G(v_i)\right) = 4n^2 - n - 6m$$

$i \neq j$

$$\sum_{v_i \quad v(G), x_j \quad v(X)} d_{\mu(G)}(v_i, x_j)\left(d_{\mu(G)}(v_i) + d_{\mu(G)}(x_j)\right)$$

$$= \sum_{\substack{v_i v_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} 2\left[(4n - 1) - \left(d_G(v_i) + d_G(v_j)\right) - d_G(v_i)\right] \qquad 1$$

$$+ \sum_{\substack{v_i v_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} \left[(4n - 1) - \left(d_G(v_i) + d_G(v_j)\right) - d_G(v_i)\right] \qquad 2$$

(1) ˘

$$4m(4n - 1) - 2 \sum_{\substack{v_i v_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} \left(d_G(v_i) + d_G(v_j)\right) - 1\,2 \sum_{\substack{v_i v_j \quad E(G) \\ v_i \quad V(G), x_j}}$$

$$= \sum_{\substack{d_G(v_i,v_j)=1 \\ v_iv_j \quad E(G)}} 2\left[(2n-2d_G(v_i))(2n-2d_G(v_j))\right] + \sum_{\substack{d_G(v_i,v_j)>1 \\ v_iv_j \quad E(G)}} (2n-2d_G(v_i))(2n-2d_G(v_j))$$

$$= \sum_{v_iv_j \quad E(G)} 2\left[4n^2 - 4n\left(d_G(v_i)+d_G(v_j)\right) + 4d_G(v_i)d_G(v_j)\right]$$

$$+ \sum_{v_iv_j \quad E(G)} \left[4n^2 - 4n\left(d_G(v_i)+d_G(v_j)\right) + 4d_G(v_i)d_G(v_j)\right]$$

$$= 8mn^2 - 8nM_1(G) + 8M_2(G) + 4n^2(C_n^2 - m) - 4n\overline{M_1}(G) + 4\overline{M_2}(G)$$

$$= 2n^4 - 2n^3 - 4mn^2 + 8mn - (4n+2)M_1(G) + 4M_2(G)$$

2 u v 2 Schultz

$$\sum_{\{x_i,x_j\} \quad v(X)} d_{\mu(G)}(x_i,x_j)\left(d_{\mu(G)}(x_i)d_{\mu(G)}(x_j)\right) = \sum_{\{x_i,x_j\} \quad v(X)} \left[2n-1-d_G(v_i)\right]\left[2n-1-d_G(v_j)\right]$$

$$= \sum_{\{x_i,x_j\} \quad v(X)} (2n-1)^2 - (2n-1)\left(d_G(v_i)+d_G(v_j)\right) + d_G(v_i)d_G(v_j)$$

$$= 2n^4 - 4n^3 + \left(\frac{5}{2} - 4m\right)n^2 + \left(6m - \frac{1}{2}\right)n + 2m^2 - 2m - \frac{1}{2}M_1(G)$$

3 u v 3 Schultz

$i = j$

$$\sum_{v_i \quad v(G), x_j \quad v(x)} d_{\mu(G)}(v_i,x_j)\left(d_{\mu(G)}(v_i)d_{\mu(G)}(x_j)\right) = \sum_{v_i \quad v(G)} \left(2n-2d_G(v_i)\right)\left(2n-1-d_G(v_i)\right)$$

$$= \sum_{v_i \quad v(G)} (4n^2-2n) - (6n-2)d_G(v_i) + 2d_G^{\,2}(v_i)$$

$$= 4n^3 - 2n^2 - 12mn + 4m + 2M_1(G)$$

$i \neq j$

$$\sum_{v_i \quad v(G), x_j \quad v(X)} d_{\mu(G)}(v_i,x_j)\left(d_{\mu(G)}(v_i)d_{\mu(G)}(x_j)\right)$$

$$= \sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} 2\left[\left(2n-2d_G(v_i)\right)\left(2n-1-d_G(v_i)\right)\right] \qquad 3$$

$$+ \sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} \left(2n-2d_G(v_i)\right)\left(2n-1-d_G(v_i)\right) \qquad 4$$

3

$$2m(8n^2-4n) - 4n\sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} d_G(v_i)+d_G(v_j) + (2-4n)\sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} d_G(v_i) + 4\sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} d_G(v_i)d_G(v_j)$$

$$= 16mn^2 - 8mn + (2-12n)M_1(G) + 8M_2(G)$$

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$$2(4n^2-2n)(C_n^2-m) - 2n\sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} d_G(v_i)+d_G(v_j) + (2-2n)\sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} d_G(v_i) + 2\sum_{\substack{v_iv_j \quad E(G) \\ v_i \quad V(G), x_j \quad V(X)}} d_G(v_i)d_G(v_j)$$

$$= 4n^4 - 6n^3 + (2-20m)n^2 + 20mn + 8m^2 - 4m + (6n-4)M_1(G) - 4M_2(G)$$

4 u v 4 Schultz

$$\sum_{v_i \quad V(G), z \quad Z} d_{\mu(G)}(v_i,Z)\left(d_{\mu(G)}(v_i)d_{\mu(G)}(Z)\right) = \sum_{v_i \quad V(G), z \quad Z} 1\left(2n-2d_G(v_i)\right)n = 4n^2 - 4mn$$

5 u v 5 Schultz

$$\sum_{x_i \quad V(X), z \quad Z} d_{\mu(G)}(x_i,Z)\left(d_{\mu(G)}(x_i)d_{\mu(G)}(Z)\right) = \sum_{x_i \quad V(G), z \quad Z} 2\left(2n-1-2d_G(v_i)\right)n = 4n^3 - 2n^2 - 4mn$$

Mycielski $\mu(G)$ Schultz

$$S^{*}\left(\mu(G)\right) = 8n^{4} - 4n^{3} + \left(\frac{9}{2} - 12m\right)n^{2} + \left(6m - \frac{1}{2}\right)n + 10m^{2} - 2m - \left(10n + \frac{5}{2}\right)M_{1}(G) + 8M_{2}(G)$$

Mycielski Schultz Schultz

1 i $S\left(\mu(K_n)\right) = 3n^3 + 11n^2 - 4n, (n \geq 2)$;

ii $S\left(\mu(K_{s,t})\right) = 8s^3 + (19t + 3)s^2 + (19t^2 + 2t - 1$

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Results on Some Indices of Complements of Mycielski Graphs

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Abstract Topology index is a mathematical descriptor of molecular structure, which digitizes the structural characteristics of molecules such as shape, size, and branching. It is easy to calculate, has objective values, and is not easily limited by experience and experiments. The study of topological index graph invariants is currently one of the most active research areas in chemical graph theory, which can be used to describe and predict the physicochemical or pharmacological properties of organic compounds. This article studies two types of degree distance metrics for the complement of Mycielski graphs: Schultz index and modified Schultz index. At the same time, expressions for the Lanzhou index of Mycielski graphs and their complement graphs of some special graphs are also provided.

Keywords Mycielski Graph Schultz index Modified Schultz index Lanzhou index